Engineering Notes

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Two-Dimensional Model for Thermal Compression

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IN Ref. 1 the simplified model (Fig. 1 of Ref. 1) used in the analysis of the effects of thermal compression on the performance of hypersonic ramjets depicted the compression, combustion, and expansion processes as one-dimensional flows. This method is commonly used for obtaining performance estimates and general operating characteristics of airbreathing propulsion devices. It avoids the difficulties of defining the flow structure within each process but leaves open the question of its validity. In the absence of experimental verification, one must resort to analytical methods which are less restrictive; e.g., one may compare results with those for two-dimensional processes, to establish whether or not the one-dimensional modeling is cogent. In this Note a two-dimensional model is introduced to demonstrate that the essential compatibility conditions of balanced pressure and aligned velocity vectors on the streamline dividing the two flows required in the one-dimensional modelling can be maintained in a two-dimensional flow. Simple oblique shocks are used to depict the inlet compression process, whereas the concept of an oblique planar heater (OPH) is used to provide a two-dimensional model of the heat addition process.

An OPH is defined as an infinitesimally thin flame front in which heat is added and the velocity vector is changed (Fig. 1). Only the so-called "weak" (Ref. 2) OPH case, in which the planar heater angle θ, with respect to the upstream velocity vector $\mathbf{U}_1$, is less than the upstream Mach angle, $\mu = \arcsin(1/M_1)$, and $\mathbf{U}_{1n}$ is subsonic, will be discussed. The "strong" OPH with $\theta > \mu$ and U_{1n} supersonic is conceivable, but since this requires a positive turning of the flow (counterclockwise in Fig. 1) it may require the coexistence of a coupled oblique shock. Tables of properties of the combined oblique shock and OPH case are given in Refs. 3 and 4. For an ideal gas, the following governing equations hold for the OPH:

$$\tan(\theta + \delta) = (1 + \gamma M_1^2 \sin^2\theta)/(1 + \gamma)M_1^2 \sin\theta \cos\theta \quad (1)$$

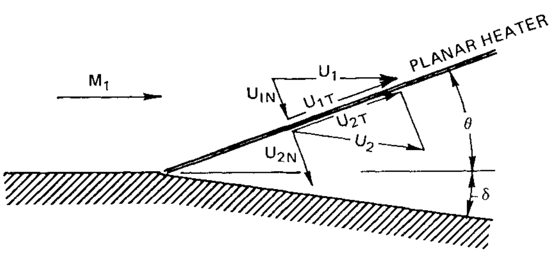

Fig. 1 Schematic illustration of oblique planar heater.

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$$p_2/p_1 = (1 + \gamma M_1^2 \sin^2\theta)/[1 + \gamma M_2^2 \sin^2(\theta + \delta)] \quad (2)$$

and

$$T_2/T_1 = (p_2/p_1)^2 M_2^2 \sin^2(\theta + \delta)/M_1^2 \sin^2\theta \quad (3)$$

where the p's and T's are static pressures and temperatures upstream (subscript 1) and downstream (subscript 2) of the OPH. In this discussion, the further restriction of the so-called "oblique Chapman-Jouget condition" (Ref. 2) will also be met; i.e., the normal component of the Mach number downstream of the OPH $[M_2 \sin(\theta + \delta)]$ will be set equal to one.

The sketch shown in Fig. 2 of the inlet compression and heat addition processes in a two-stream thermal compression model is drawn to scale for a particular case with the following conditions: $M_0 = 5$, $\gamma = 1.4$, and an initial wedge angle on the lower compression surface (δ_{0-1}) of 10°, followed by a $-8°$ surface ($\delta_{1-2} = 18°$). The choice of the number of shocks, OPHs and geometry is arbitrary and, indeed, to match the performance indicated in the one-dimensional analysis, the compression surfaces would have to be more complex, e.g., double rather than single wedges. Even for the relatively simple model shown, the calculations are involved, due to the iterative nature of the solution. Air in the primary streamtube is compressed through shocks 0-1 and 1-2; sufficient heat is added in OPH 2-3 to yield compatibility in pressure and flow direction between regions 3 and 4. This condition is obtained by the simultaneous solution of the OPH equations in the primary streamtube for OPH_{2-3} and the shock equations in the secondary streamtube for the shock 0-4′. A second OPH turns the flow in the primary streamtube so that it is coaxial (i.e., $\mathbf{U}_5$ is parallel to the freestream velocity vector $\mathbf{U}_0$). After passing through the initial shock, air in the secondary streamtube is turned coaxial ($\mathbf{U}_7$ parallel to $\mathbf{U}_0$ and $\mathbf{U}_5$) by a cowl-reflected shock and OPH_{6-7}. The strength of the cowl-reflected wave is set by the pressure compatibility condition that must be met in regions 5 and 7. The compatibility conditions for flow direction and pressure between regions 3 and 4 and regions 5 and 7 are more easily identified in the polar plots of the OPH and shock processes shown in Fig. 3. Note that for this particular example, shock 4-6 is quite weak; thus, the internal cowl angle is nearly as large as the initial turning angle $\delta_{0'-4}$ in the secondary streams. Although pressure and flow direction are matched along the dividing streamline, the entropy downstream of the point of initial coalescence of the three shocks and OPH_{2-3} is not; thus, the streamline must, in fact, be a vortex sheet.

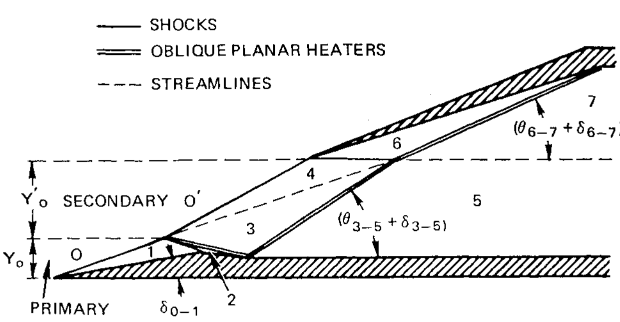

Fig. 2 Two stream model for thermal compression analysis.

Table 1 Flowfield properties

Region	Turning angle, deg, $\delta_{(n-1)-n}$	Shock angle, deg, $\theta_{(n-1)-n}$	Flow direction, deg	Mach Number	Static pressure ratio, p/p_0	Static temp. ratio T/T_0	Total temp. ratio, T_t/T_{t0}
0	0	0	0	5	1.0	1.0	1.0
1	10	19.376	+10.000	3.999	4.392	1.429	1.0
2	18	30.239	−8.000	2.172	13.896	2.428	1.0
3	26.644	5.683	+18.644	1.870	6.375	7.082	2.006
4	18.644	28.266	+18.644	3.151	6.375	2.010	1.0
5	18.644	14.267	0	1.840	3.466	9.744	2.724
6	1.184	19.312	+17.460	3.086	7.011	2.065	1.0
7	17.460	6.662	0	2.447	3.446	3.891	1.425

The mass flow ratio (i.e., the split) for the streams is obtained from the geometry:

$$\frac{Y_0'}{Y_0} = \left[\frac{\sin(\theta_{0-1} - \delta_{0-1})\sin(\theta_{1-2} - \delta_{1-2})}{\sin\theta_{0-1}\sin\theta_{1-2}\sin\theta_{2-3}}\right] \left[\cos(\theta_{2-3} + \delta_{1-2} - \delta_{0-1}) + \frac{\cos(\theta_{3-5} + \delta_{3-5})\sin(\theta_{2-3} + \delta_{2-3})}{\sin\theta_{3-5}\sin\theta_{2-3}}\right] \Bigg/ \cot\theta_{0'-4} + \frac{\cos(\theta_{4-6} - \delta_{0'-4})\sin(\theta_{0'-4} - \delta_{0'-4})}{\sin\theta_{4-6}\sin\theta_{0'-4}}$$

where δ is the turning angle and θ is the shock (or OPH) angle measured from the upstream velocity vector. The double subscripts refer to the regions upstream and downstream of the shock or OPH corresponding to Fig. 2.

To enable the reader to check the example case, the flow properties in each of the regions are listed in Table 1. Note that there is considerably more heat addition in the primary streamtube, as indicated by the value of 2.724 for T_t/T_{t0} in region 5 vs 1.425 in region 7. In this example, the maximum internal contraction ratio, $A_0/A_d = 1.204$ (ratio of captured streamtube height to minimum internal duct height), occurs in the plane of the cowl lip. On the other hand, if the dividing streamline were replaced by a solid surface which would be required to obtain the same performance in the absence of the beneficial effect of thermal compression, the respective A_0/A_d values would be much greater, i.e., $A_0/A_d = 2.036$ in the primary, and $A_0/A_d = 3.012$ in the secondary.

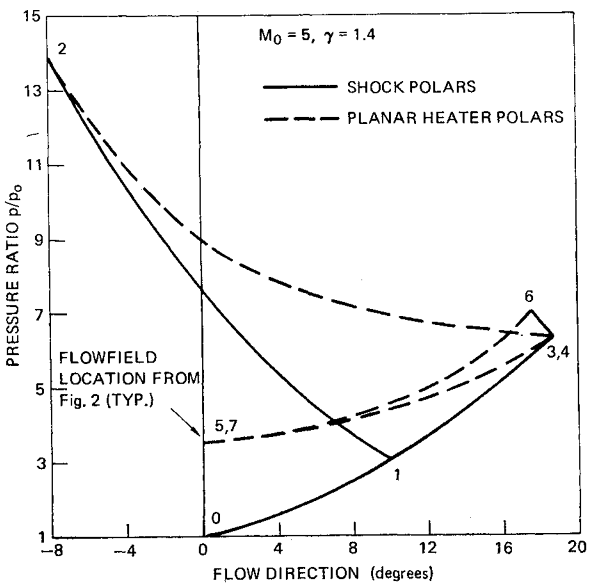

Fig. 3 Oblique planar heater and shock polars.

Consistent with the intent in the original paper,[1] the relatively simple gasdynamic models for thermal compression are used here to facilitate the basic understanding of the concept. The very formidable problem of developing a fuel distribution system for a realistic engine design that would place planar flame fronts in the desired locations is beyond the scope of this discussion.

References

[1] F. S. Billig, R. C. Orth, and M. Lasky, "Effects of Thermal Compression on the Performance Estimates of Hypersonic Ramjets," *Journal of Spacecraft and Rockets*, Vol. 5, No. 9, Sept. 1968, 1076–1081.

[2] F. S. Billig, "External Burning in Supersonic Streams," TG 912, May 1967, Applied Physics Lab., The Johns Hopkins Univ., Silver Spring, Md.

[3] H. W. Woolard, "Tables of Properties of Some Oblique Deflagrations in Supersonic Flow," TG-382, Sept. 1960, Applied Physics Lab., The Johns Hopkins Univ., Silver Spring, Md.

[4] W. Chinitz, L. C. Bohrer and K. M. Foreman, "Properties of Oblique Detonation Waves," AFOSR TN 59-462 (ASTIA AD 215-267), April 15, 1969, Fairchild Engine Div., Deer Park, N. Y.

Aerospace Application of Atmospheric Rendezvous

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Introduction

THERE are a number of situations in which an atmospheric form of rendezvous has been used in aeronautics (see Fig. 1). Space rendezvous has also been used to increase the efficiency and flexibility of operations in the Apollo manned lunar mission. An examination of aerospace missions of present interest indicates that orbital logistics and hypersonic flight vehicles may benefit in efficiency and flexibility of operation by the use of an atmospheric rendezvous. The

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